GPS-GAP: A Post Baccalaureate Certificate Program[1]

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Abstract: The University of Maine has been offering a Post-Baccalaureate Certificate of Completion program in GPS-GAP (GPS, Geodesy and Application Program) since 2005. The undergraduate and graduate courses are available asynchronously via the Internet using WebCT technology. The courses can either be taken for credit toward an academic degree, or as continuing education credits. The course material is mathematically based, and algorithmically and geodetically correct and complete. Asynchronous and iterative learning strategies tightly integrate the textbook, PowerPoint presentations with audio, streamed lectures, Mathcad solutions with audio, processing of pseudorange and carrier phase observations, and last but not least quiz questions. As a result of this approach, the class size is effectively 1 (one). Utilizing the iterative learning strategy and free communication with Skype, the ideal level of individualized instruction is achieved with the extra benefit of being location-independent as to student or instructor. This unified and innovative model of optimal learning is attractive and beneficial not only to traditional college students, but also to practicing surveyors and engineers by conveying knowledge at a depth that was difficult to achieve thus far. Although the roots of GPS-GAP are a quarter of a century old, focusing on fundamentals, carefully choosing collaborative efforts and models of cooperation promises a bright future for this program.

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Introduction

Geodesy has been driven by cycles of discovery and increased measurement accuracy ever since the determination of "the size of the earth" by Eratosthenes of Cyrne's in ancient times. Surely, throughout history, whenever a new cycle began, there were those who feared the "end had been reached", and yet, the cycles continued. Such angst surfaced in the early nineteen eighties when the GPS surveying cycle began with GPS receivers weighing 100 pounds or so, and we no longer needed to construct observation towers. The current cycle involves two mega trends that we cannot ignore: nation-wide network RTK and broadband internet access to every household. GPS-GAP has been designed to function within this cycle.

We will address the roots of GPS-GAP, its content, some key characteristics, and current implementation.

The Roots of GPS-GAP

The roots of GPS-GAP are traceable to experiences gathered on top of Mount Watchusett, at late evening hours during the summer of 1982 while testing the prototype Macrometer GPS receiver developed at M.I.T. The satellite visibility ranged from about 6 pm to midnight in New England. Many of the sunset watchers at the summit were puzzled by my activities and impressed by the huge piece of equipment in the back of the station wagon, the abundance of cables, and the strange looking antenna (so they thought). Their puzzlement about what I was up to was reflected in some of their comments, such as "Is this thing taking off?", or "Are you on our side?"

Of course, there was plenty of time until midnight to marvel about Fourier transforms flashing on the computer screen, the amazing precise nightly repeatability of the 30 km baseline vector to Woburn at the northern edge of Boston, and, generally, at the seemingly unlimited potential of GPS. Luckily, my excitement about GPS was so elevated that I did not even pursue the re-occurring thought that GPS might be dangerous to my career in geodesy, which I had barely began. There certainly was plenty of curiosity, e.g. what signals did the satellites actually transmit, how could one get something useful out of these signals since they were supposedly below the background noise, or why was the computer crunching so furiously and continuously all night.

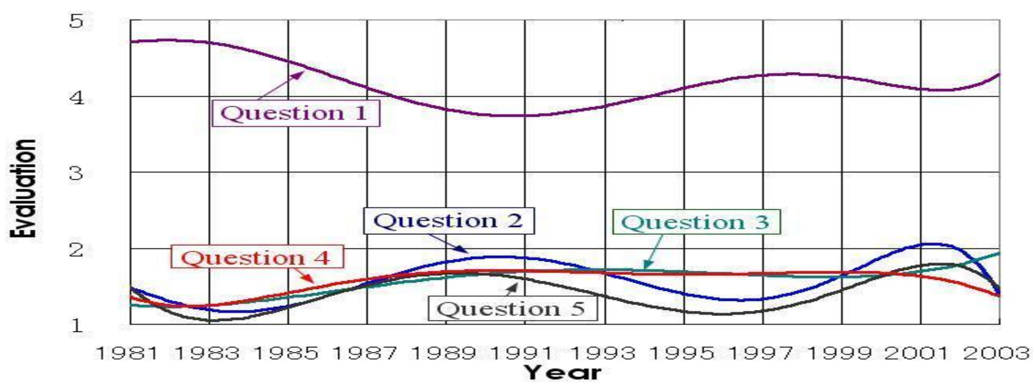
I knew enough about satellite orbital determination from the college days to understand that the observations on top of Mount Watchusett by themselves were insufficient to estimate satellite orbital parameters, in addition to baseline vector components. A global network of tracking stations had to be used. However, how did observations from such a network arrive at the processing center? I was actually using the BITNET, the immediate forerunner of the Internet, to supervise a graduate student in

Maine while at M.I.T that summer. This early BITNET experience not only resolved for me the puzzle of global data transmission, but also gave me a first glimpse of its potential for distance education.

The amazement with the science underlying GPS satellite surveying made me rush to establish a graduate course in GPS at the University of Maine that Fall of 1982. The urge to tell the GPS story propagated into three editions of the book *GPS Satellite Surveying* and now into the series of GPS-GAP Internet courses.

Mostly, the GPS-GAP courses are updated versions of two undergraduate courses SIE 401 (Adjustment Computations) and SIE 451 (Geodetic Models), and one graduate course SIE 541 (Satellite Positioning). These courses had been at the core of the Surveying Engineering program for about 25 years. The averaged student evaluation shown in Figure 1 indicates that the workload for these courses was high compared to others in the department. The reason probably was the mathematical rigor by which the material was presented. Students often remarked in appreciation, "I finally understand why we had to take all that math". Other students, often those with significant surveying experience, frequently commented as to the contents, "I had no idea that something like this existed".

As a teacher, I am particularly proud of having received a high evaluation on the questions "encouragement to think for themselves" and "intellectual discipline required". I have made every effort to assure that GPS-GAP students feel the same.



Q 1: Work load compared to others courses? (*1-lighter, 5-much heavier*)
Q 2: Enthusiasm displayed by the Instructor? (*1-very much, 5-very little*)
Q 3: Encourage students to think for themselves ? (*1-very much, 5-very little*)
Q 4: Intellectual discipline required? (*1-very much, 5-very little*)
Q 5: Instructor's confidence in his knowledge? (*1-very much, 5-very little*)

Figure 1: Student Evaluations for SIE 401, 451 and 541.

GPS-GAP Content

Looking at the rapid developments in GPS satellite surveying, we observe a confluence of trends that is leading to some stabilization soon. The evolution of GPS surveying is likely to converge toward real-time network RTK. As is well known, there is little for GPS positioning solutions to improve beyond ambiguity fixed solutions. It is a natural wall that is preset by the carrier frequencies. The modernized GPS satellites and other satellite systems such as GLONASS, Galileo and Compass will make ambiguity fixing more reliable and faster to achieve, but there will not be much, if any, gain in positional accuracy at the centimeter level.

In terms of electronic communication, in the near future every household, and thus every student, will have broadband access to the internet, at least at a capacity now achieved by TV cables. This will open up a new way of instruction, i.e. individualized instruction in which the class size is one (1). I cannot imagine a more focused approach to learning.

In recognition of these "mega trends," GPS-GAP will provide the foundation needed for successfully operating in such a fully developed surveying environment, and do so by focusing on individualized instruction of students that is truly location-independent.

When putting together the GPS-GAP content we recognized that the human mind is naturally inquisitive, and that pushing buttons on any instrument and running any single-purpose commercial software is generally not enlightening. We further acknowledged, whether one likes to hear it or not, that surveying is deeply entrenched in mathematics and physics. Counting on this natural desire to learn and to know, we perceive a need for an educational service that embraces the fundamentals of GPS surveying.

What are these fundamentals?

Looking in our surveying backyard, we recognize the SPC (State Plane Coordinate) systems as a signature creation that supposedly "makes it all so easy for surveyors." The mathematical foundation is undeniable: Cauchy-Riemann Equations, Gauss first and second fundamental coefficient and many more elements from differential geometry, geodesic on a curved surface, conformal mapping of a curved surface, and so on. It is hard to argue that this is not pretty steep mathematics. Of courses, there is the need to connect state plane coordinates to the outcome of a GPS survey.

We do not have to look much farther to find another example. The geoid, which is still the most popular reference surface for heights, is an expression of the earth's gravitational and centrifugal potential. In order to match the accuracy of measuring ellipsoidal heights with GPS, the determination of the accurate geoid is still a mathematical challenge, which requires gravity observations, altimeter observations from satellites, and analysis of satellite orbital perturbation. Again, plenty of mathematics is necessary.

Of course, our stations are located on a deformable earth. Indeed, just a little digging reveals the mathematical and physical elements needed to formulate workable solutions. An immediate implication for surveying is that our terrestrial and celestial reference frames, i.e. the ITRFs and ICRFs become a function of time. If we ever hope to fully harvest the coordinate production capability of GPS, or the GNSS (Global Navigation Satellite System), these temporal reference frame variations cannot be ignored. Again, an understanding of many fundamentals from mathematics and physics is important.

It is really not necessary to escape into the "realm of geodesy" to find justification for using mathematics, knowing well that the delineation between geodesy and surveying is murky at best. "Surveying by any measure" has its share of mathematical depth. Examples are linearization of multi-dimensional multi-variable functions in adjustments, multivariate normal distributions, derivation of statistical tests, and constrained estimation for General Linear Hypothesis (GLH) testing, generalized matrix inverses for inner constraint solutions, etc.

Last, but not least, there are the fundamentals of GPS satellite surveying. Some of them include undifferenced solutions (navigation solution), and single, double and triple differencing of the observables, network RTK, and PPP (Precise Point Positioning). Environmental impacts of the ionosphere and troposphere, and signal multipath are important. The ambiguity fixing technique LAMBDA offers great insight into the role of correlations between estimated parameters in connection with general linear hypothesis testing.

GPS-GAP Points of Interest

GPS-GAP has been designed as a cost effective approach to education that takes advantage of the Internet. The courses are offered asynchronously to individuals (class size =1), taking advantage of free computer to computer audio communication, and having flexible starting and ending times. The student selects the most convenient study times, and proceeds at a pace that fits the needs of the individual. All courses have been designed as 1-credit hour units, giving students greater freedom to select courses according to prior knowledge of the subject. A broadband Internet access and an Internet browser is all that are necessary to take the courses. The following are some distinguishing characteristics of GPS-GAP:

Mathematical Depth. Modern programs such as Mathcad make it possible to teach and experience mathematics in great depth. They help in comprehending mathematical derivations and in verifying and visualizing expressions and equations instantly. Mathcad has been specifically designed for easy programming of mathematical content, and, most importantly, it can be learned "on-the-fly". Mathcad adds a new dimension to the learning process, and has become a major pillar in the GPS-GAP learning strategy.

Understand the Physics. At the level of a centimeter the earth behaves like a deformable body. Material that needs to be considered includes plate tectonic motions, solid earth tides, ocean loading, and polar motion. Because of the deformable property of the earth, special attention must be given to the definition of the reference frames (ITRF, ICRF), the geodetic datum, and how observations are linked to model observations. Because of GPS signal path, we must also be concerned with the troposphere and the ionosphere. In GPS-GAP, therefore, we address the physics of the earth in detail, starting at the geocenter and ending at the upper edge of the ionosphere.

Satellite Transmissions and Signal Modulations. The selection of satellite transmissions impacts the receiver capability, and therefore the availability of the user to meet project specifications. The educated GPS user should understand the implications of the C/A code and P-Code modulations on positional accuracy, and the difference between broadcast and precise ephemeris. Such a user should further be able to follow ongoing efforts to modernize GPS, understand the additional measures that must be taken when combining GPS and GLONASS observations, or even more generally, understand compatibility issues between satellite systems. In GPS-GAP, we will pay detailed attention to these aspects.

Pseudorange and Carrier Phase Observables. In GPS-GAP we consider baseline vectors or point positions to be derived quantities, and develop all algorithms and software to compute vectors and positions, starting with carrier phases and pseudoranges.

The various satellites signals that arrive at the receiver antenna are subjected to a huge amount of processing inside the receiver. Yes, this is indeed another fruitful area for learning more about GPS. However, we short circuit that process and, instead, focus of the output of a receiver only, i.e. the pseudorange and carrier phase observables. Other GPS-GAP courses are under development which will address receiver-internal processing.

Mathematic, Geodetic, and Algorithmic Completeness. A fundamental test of completeness is that the material is presented with such rigor and depth as needed to understand the mathematical models used in processing observations. Observations include classical terrestrial angles, distances and leveling measurements, and GPS carrier phase and pseudorange observations. Special care is taken to avoid simplifications that could cause gaps in the understanding and comprehension of the mathematical models. Continuity of development is stressed when deriving expressions and algorithms.

Receiver Antenna. There exists a large variety of receiver antennas. Distinguishing characteristics of these antennas are how well they suppress signal multipath, or how much the phase centers vary with direction of the incoming signal. There is a special course dedicated to receiver antenna issues.

Avoiding Hardware. The GPS-GAP courses do not address hardware, e.g. specific receivers. There is no emphasis made on receiver characterization and performance beyond the basic classification of single versus multiple-frequency receivers. Data sets are obtained from public databases as needed or simply generated by in-house receivers. This approach makes the GPS-GAP courses hardware independent and manufacture-neutral.

In-House Software Only. All software is available in Mathcad and has been developed at the University of Maine over the last 20 years. This includes carrier phase and pseudorange processing software for baselines or precise point positioning (PPP), adjustments in one, two or three dimensions, conformal mapping, etc. We make no effort to critically evaluate software as to implementation, robustness, etc., since our focus is on mathematics, physics, and statistics.

Quiz Questions. Quiz questions are another important and integral part of the GPS-GAP distance learning strategy. The complete contents of the courses, i.e. mathematical derivations, final expressions, numerical values and considerations, theoretical concepts, figures, Mathcad source code, rules of thumb and so on, are subjects of quiz questions. All questions are presented in form of graphical images which make it possible to create powerful composites, e.g. combining equations and figures, equations and text, equations and Mathcad code, etc.

Iterative Learning Strategy. A GPS-GAP course consists of

a) Textbook (the same for all courses)

b) Course specific reference material

c) PowerPoint presentations with audio for all lectures

d) Streamed version of all lectures

e) Large number of Mathcad implementations with audio

f) Live Mathcad solutions (dedicated server)

g) Quiz questions

The quiz questions can be made available to the students prior to taking the test. This may seem like an unusual practice at a first glance; it is a defensible strategy even for the most particular academic policing, since the questions address minute detail and are randomly drawn from a very large pool of questions.

The quiz questions are effectively used in the cycle of learning that comprises yet another part of the GPS-GAP iterative learning strategy. Students can first listen to the lecture and the Mathcad implementation, perhaps even experiment with live Mathcad solutions, and then study the quiz questions. If they do not feel ready to take the quiz they can listen to the lecture again, study the quiz questions once more, and repeat this cycle until ready to take the quiz. This iteration assures that students do not overlook the mathematical detail and the finer points presented in the lecture, because the quiz questions will subtly remind them of what they have missed. This strategy has been found to be particularly successful in making the depth of the GPS-GAP courses understandable to students whose formal college days are only a distant memory.

Location-Independent Service. GPS-GAP strives to achieve location-independency as to student and instructor. The WebCT implementation is accessible to students 24/7/365. The instructor and student communicate efficiently and effectively via free computer calls, e.g. using Skype. In case a student needs clarification, both the student and the instructor sit in front of the computer and view the same PowerPoint page, Mathcad worksheet, or textbook page. In this way, efficient and effective individualized instruction can be realized.

Current Implementation

GPS-GAP uses the WebCT program to deliver the courses over the internet, using a server located at Orono and operated by the Division of Lifelong Learning. A second server is located at the Hutchinson Center at Belfast, Maine, and is exclusively dedicated to GPS-GAP and runs Mathcad live computations. There are currently 11 courses with a large database of quiz questions. Four enrolment options are available.

WebCT

Figure 2 shows a typical opening page of a GPS-GAP course. The PowerPoint presentation (PowerPoints) and the streamed version of the lecture (Impatica) also contain all Mathcad htm files with audio. Mathcad Live links to live computations. On the left there are links to the quizzes and a calendar. The other links, Chat, Mail, and Who's Online, enable communications between instructor and students.

Figure 3 shows a sample of a Mathcad implementation that uses actual GPS carrier phase and pseudorange observations. For easy navigation and integration, audio reference items are numbered and highlighted in yellow, references to PowerPoint lectures are made by copying equations (see highlight in pink), and references to the textbook are in red. There are about 5 worksheets per lecture. It is recommended that students run their own Mathcad for in-depth experimentation, building upon the many solutions provided.

Figure 2: Opening Page of a GPS-GAP Course.

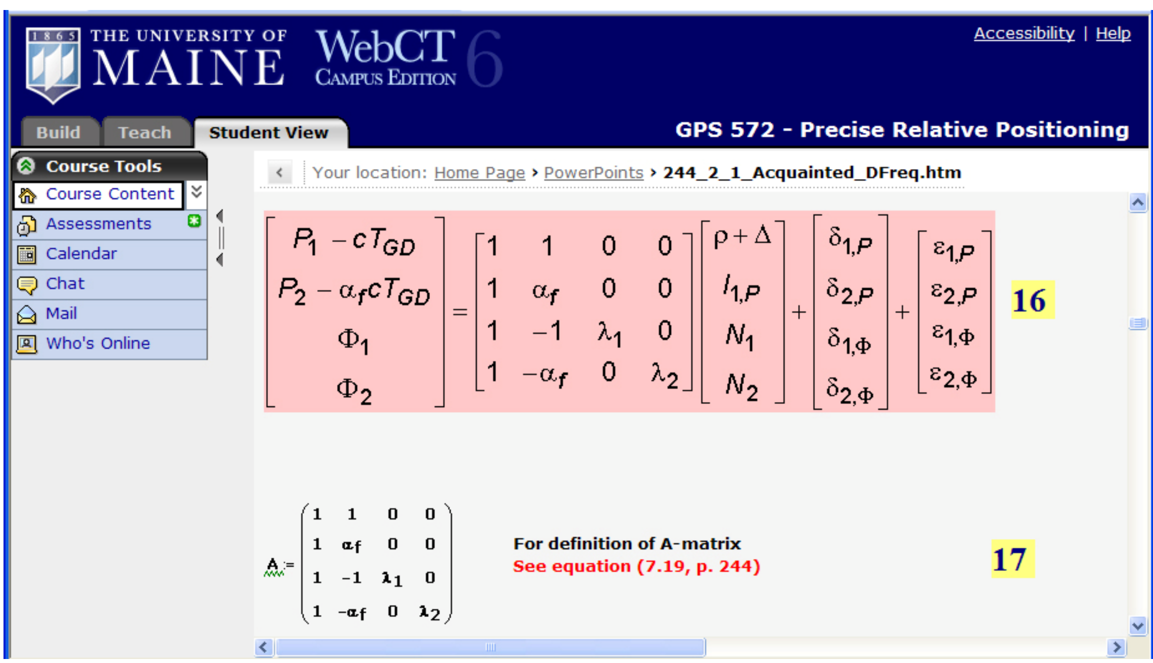


Figure 3: Example of Mathcad Worksheet.

Courses

Table 1 shows the current collection of GPS-GAP courses. All GPS courses carry 1-credit hour, whereas MAT 262 and 332 are traditional, non-distance 3-credit hour courses offered by Department of Mathematics and Statistics.

The mathematics refresher course GPS 101 is a recent addition. It serves individuals who practice surveying and engineering and would like to refresh their mathematical skills. The course addresses selective topics that are needed to understand the mathematical foundation of GPS positioning and GPS data processing, including adjustments. The material covered is typically spread over 3-4 traditional university courses. At this time the course is available only as non-credit.

The most recent addition is GPS 490 (GNSS Receiver Antenna). GNSS receiver antennas are treated from the user's point of view. The basics of electromagnetic field theory are discussed at the level necessary for understanding antenna performance parameters. Complex notation and dB scale are introduced as basic tools for treating performance. The antenna angular response patterns are presented for transmitting and receiving modes. A detailed discussion of wave propagation and reflections in transmission lines leads to the treatment of antenna mismatch and frequency response. Typical sources of noise for GNSS applications are analyzed; the propagation of signal and noise through the electronic circuits is discussed in detail. The course concludes with a section on estimating the signal to noise ratio at the receiver output. This ratio is often available to users to aid them in carrier phase processing. The course is currently available only as non-credit.

GPS 490 has been designed and is being taught by Dr. Dmitry Tatarnikov, who is a faculty member at the Moscow Aviation Institute, and is Chief Antenna Design Engineer at TOPCON, Moscow, Russia.

Assessments

Various assessment options are available to test the student's progress. All options include a set of 5 multiple choice quizzes which are computer graded. The test results are immediately returned to the student. The quizzes cover overlapping material. For example, the second quiz covers lectures 1-4, even though lectures 1 and 2 have already been tested in the first quiz. The fifth quiz covers all of the material. In the current implementation, each quiz consists of 50 randomly selected questions from a large pool of questions which must be answered in 50 minutes. See Figure 4.

When the course is taken for credit at the University of Maine, the student is required to prepare one Mathcad worksheet or submit one typed paper. The student proposes three topics with a brief description; one of them will be approved. The paper must contain a substantial amount of mathematics, or address relevant aspects from physics in detail, or amplify existing Mathcad implementations to create a new one. The report must be submitted in MS Word or converted to Acrobat pdf format. Mathcad worksheets must be amply documented.

Table 1. The Course Matrix

GPS 401 Adjustments with Obs. Eq.	GPS 402 Adjustment Algorithms	GPS 403 Quality Control with Adjustments	GPS 101 Mathematics Refresher
GPS 441 3D Geodetic Model	GPS 402 Ellipsoidal Surface Model	GPS 403 Conformal Mapping Model	
GPS 570 Fundamentals of Satellite Positioning	GPS 571 Precise Point Positioning	GPS 572 Precise Relative Positioning	*MAT 262 Liner Algebra*
GPS 490 GNSS Receiver Antenna	Software Receiver (under development)		*MAT 332 Statistics for Engineers*

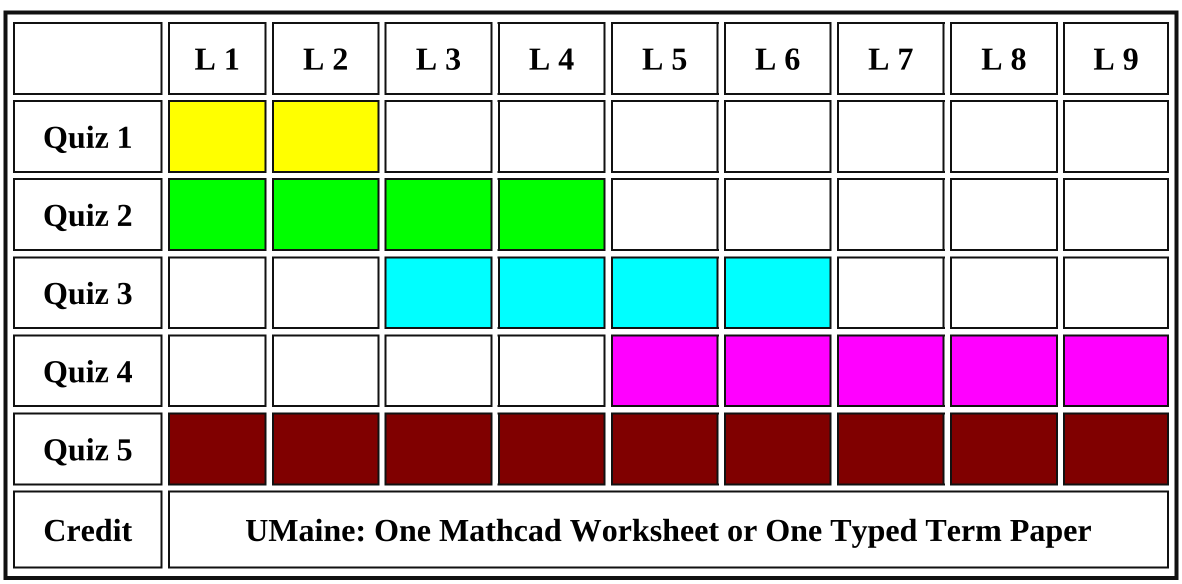


Figure 4: Course Assessment

Enrolment Options

There are several ways in which courses can be taken. It is important to note that in all case the course contents, i.e. the PowerPoint presentations, the Impatica, the Mathcad solutions, and quiz questions are the same. All courses have access to a dedicated server for real-time Mathcad computations.

Non-Credit.

1. Registration is available through www.gnss.umaine.edu
2. Registration fee is the same for all courses, whether undergraduate or graduate.
3. Passing score is 60% on quizzes.
4. Separate Mathcad solution or term paper is not required
5. The course can start or end at the convenience of the student
6. Each course earns 1.5 continuing education credits (CEU)

Credit at UMaine:

1. Registration is available through www.gnss.umaine.edu
2. UMaine tuition rules apply. See www.umaine.edu/bursar
3. UMaine course grading policies apply
4. A Mathcad solution or paper is required
5. UMaine semester schedule applies
6. No CEUs given

Credit at student's home university: *group of students*

1. Registration is arranged by the Division of Lifelong Learning
2. Fee is negotiated
3. Home university's grading policies apply
4. Mathcad solution or paper are optional
5. Follow home university's schedule
6. Credit will be given at home institution

Credit at student’s home university: *individual student*

1. Register for the non-credit version at UMaine
2. Non-credit fee applies.
3. Passing score is 60% of all quizzes (UMaine)
4. Register for special topics course at home university
5. Student advisor (home university) maintains advising role
6. Student advisor supervises paper (if necessary)
7. Follow home university’s time schedule
8. The student advisor assigns the final grade
9. Credit is given at home institution

Conclusion

The key element of GPS-GAP is that the courses are mathematically based and have a geodetic slant. The courses are available asynchronously and the delivery mode takes advantage of the Internet to achieve individualized instruction. GPS-GAP seeks to serve a national or even international audience of learners, realizing GPS and geodetic education without borders. Cooperation with other teachers and researcher is welcomed to update, maintain, and expand the number of courses.

Appendix: UMaine Catalog Descriptions

GPS 101 Mathematics Refresher (currently offered as non-credit only): Real and complex number systems and operations, basic functions, linear algebra with emphasis on matrices, eigenvalues, eigenvectors, differentiation, integration, elements of vector calculus, spherical trigonometry, interpolation, solution of linear and non-linear equation systems, quadratic forms, minimization of multi-dimensional functions, basic univariate distributions and functions thereof, and multivariate normal distribution.

GPS 401 Adjustments with Observation Equations: Errors, stochastic and mathematical models, quadratic forms, linearization and variance-covariance propagation of multi-dimensional nonlinear functions, least-squares algorithm of observation equations, position estimation using surveying and GPS vector measurements that are nonlinear functions of parameters; review of statistics and linear algebra. Prerequisites: MAT 262, MAT 332, equivalent or consent. Cr 1

GPS 402 Adjustments Algorithms: Error ellipses and ellipsoids, propagation of estimated quantities, a priori information on parameters, adjustment of implicitly related observations and parameters, mixed model, condition equation model, sequential solutions, testing conditions on nonlinear parametric functions. Prerequisites: GPS 401, equivalent or consent. Cr 1

GPS 403 Quality Control with Adjustments: Geometry of least-squares, definition of network coordinate systems, singularities, probability regions, minimal and inner constraints, invariant quantities, multivariate normal distribution, relevant statistical tests, type I/II errors, internal and external reliability, absorption of errors, blunder detection, decorrelation, inversion of patterned and large matrices, numerical aspects; Kalman filtering. GPS 401, equivalent or consent. Cr 1

GPS 441 Three-Dimensional Geodetic Model: Conventional celestial and terrestrial references frames, precession, nutation, polar motion, geodetic datum, geoid, ellipsoid of revolution, geodetic coordinates, height systems, 3D geodetic model and model observations, reduction of observations, observation equations, partial derivatives, 3D network adjustments, height-controlled 3D networks, GPS vector observations, review of spherical trigonometry and spherical harmonic expansions. Prerequisite: GPS 401, GPS 403, equivalent or consent. Cr. 1

GPS 442 Ellipsoidal Surface Model: Geodesic line on the ellipsoidal surface, geodesic curvature, differential equations of the geodesic, direct and inverse solutions, 2D network adjustment on the ellipsoidal surface, partial derivatives, reduction of observations, traditional horizontal and vertical networks in surveying and geodesy; in-depth review of differential geometry. Prerequisite: GPS 441, equivalent or consent. Cr. 1

GPS 443 Conformal Mapping Model: Conformal mapping of the ellipsoidal surface, meridian convergence, point scale factor; State Plane Coordinate systems, Transverse Mercator, Equatorial Mercator, Lambert Conformal with one or two standard parallels, polar azimuthal, and UTM; reduction of observations, computations on the conformal map and relation to the surface of the earth; review of complex variables. Prerequisite: GPS 441, GPS 442, equivalent or consent. Cr. 1

GPS 490 GNSS Receiver Antenna (currently offered as non-credit only): Basics of electromagnetic waves, polarization, antenna angular response pattern and gain, polarization properties of GNSS user antennas, phase pattern, phase center variations and antenna calibrations, carrier phase multipath, reflections from the underlying terrain, antenna down/up ratio, basics of transmission lines, antenna mismatch and frequency response, cable losses, noise propagation and signal-to-noise ratio.

GPS 570 Fundamental of Satellite Positioning: ITRF and ICRF references frames and transformations, tectonic plate motions, precession, nutation, polar motion, rotational and atomic time scales, GPS time, normal orbits, Kepler's laws and equation, topocentric satellite motions, visibility, perturbation of satellite orbits, solar radiation pressure, impact of asymmetry of gravity field and earth's flattening; GPS, GLONASS and Galileo satellite systems. Prerequisite: GPS 401 and 441, MAT 262 and 332, equivalent, or consent. Cr. 1

GPS 571 Precise Point Positioning: Pseudorange and carrier phase observables, satellite time, relativity, broadcast and precise ephemerides, range iteration, receiver and satellite clock errors; singularities, tropospheric refraction and absorption, impact of the ionosphere, solid earth tides, ocean loading, satellite antenna offset, phase windup correction, closed form solutions; Kalman filter; timing, mapping of the spatial and temporal variation of the troposphere and ionosphere. Prerequisite: GPS 401 and 441, MAT 262 and 332, equivalent, or consent. Cr. 1

GPS 572 Precise Relative Positioning: Differencing observables in space and time, common-mode error reduction, geometry-free solutions, widelaning, closed-form solutions, cycle slips, constraint solutions, integer ambiguity estimation, LAMBDA, antenna calibration, multipath on pseudoranges and carrier phases, spatial vector networks, differential corrections, global data collection and maintenance, GPS services. Prerequisite: GPS 401 and 441, MAT 262 and 332, equivalent, or consent. Cr. 1

MAT 262 Linear Algebra: An introduction to matrices, systems of linear equations, linear transformations, determinants vector spaces, orthogonality, eigenvalues and eigenvectors, with applications. Some use will be made of mathematical software. Because of overlap, MAT 258 and MAT 262 cannot both be taken for degree credit. Cr. 3

MAT 332 Statistics for Engineers: Statistical methods applicable to engineering including theory and application of classical and nonparametric methods. Prerequisite: MAT 228. Cr. 3